

Work and Energy

Name: _____

1. A spring has a spring constant of $k = 55.0 \text{ N/m}$. The spring is compressed a distance of 3.50 cm . What is the potential energy stored in the spring?

$$PE_{\text{spring}} = \frac{1}{2} kx^2 = 0.5 \cdot 55.0 \text{ N/m} \cdot (0.0350 \text{ m})^2 = 0.0336875 \text{ J} = \boxed{0.0337 \text{ J or } 33.7 \text{ mJ}}$$

2. An 85 g wooden block is set up against a spring. The block rests on a smooth surface. The block is pushed into the spring, compressing it a distance of 2.0 cm and then released. The spring constant is $k = 78 \text{ N/m}$. What is the speed of the block when it reaches its initial position (where the spring was not compressed)?

$$\begin{aligned}\sum E_i &= \sum E_f \\ \frac{1}{2} kx^2 &= \frac{1}{2} mv^2 \\ 0.5 \cdot 78 \text{ N/m} \cdot (0.020 \text{ m})^2 &= 0.5 \cdot 0.085 \text{ kg} \cdot v^2 \\ v^2 &= (78 \text{ N/m} \cdot 0.0004 \text{ m}^2) / 0.085 \text{ kg} = 0.3670588235 \text{ N}\cdot\text{m}/\text{kg} \text{ (since } \text{N}=\text{kg}\cdot\text{m}/\text{s}^2, \text{N}\cdot\text{m}/\text{kg} = \text{m}^2/\text{s}^2) \\ v &= 0.605853797 \text{ m/s} = \boxed{0.61 \text{ m/s}}\end{aligned}$$

3. A roller coaster starts at some height that you do not know. It goes down this hill and then goes up a second hill that is 28.5 m above the first drop at a speed of 22.5 m/s . So how high was the initial hill?

$$\begin{aligned}\sum E_i &= \sum E_f \\ mgh &= \frac{1}{2} mv^2 \\ gh &= \frac{1}{2} v^2 \\ 9.8 \text{ m/s}^2 \cdot h &= 0.5 \cdot (22.5 \text{ m/s})^2 \\ h &= (0.5 \cdot 506.25 \text{ m}^2/\text{s}^2) / 9.8 \text{ m/s}^2 = 25.82908163 \text{ m} = \boxed{25.8 \text{ m}}\end{aligned}$$

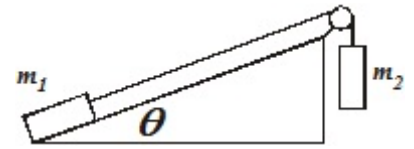
4. A 5.0 kg crate slides down a smooth ramp that is elevated at an angle of 38° . The length of the ramp is 2.0 m . What will be the speed of the crate at the bottom of the ramp?

$$\begin{aligned}h &= 2.0 \text{ m} \cdot \sin(38^\circ) = 1.231322951 \text{ m} \\ \sum E_i &= \sum E_f \\ mgh &= \frac{1}{2} mv^2 \\ gh &= \frac{1}{2} v^2 \\ 9.8 \text{ m/s}^2 \cdot 1.231322951 \text{ m} &= 0.5 \cdot v^2 \\ v^2 &= (9.8 \text{ m/s}^2 \cdot 1.231322951 \text{ m}) / 0.5 = 24.13392983 \text{ m}^2/\text{s}^2 \\ v &= 4.912629625 \text{ m/s} = \boxed{4.91 \text{ m/s}}\end{aligned}$$

5. A fireman runs up a 7.5 m ladder. The fireman has a mass of 52 kg and is carrying 15 kg of firefighting gear. If the fireman developed 685 watts , how much time did it take to reach the top of the ladder?

$$\begin{aligned}P &= W/t \text{ so } t = W/P = (F \cdot d)/P \\ t &= (67 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 7.5 \text{ m}) / 685 \text{ W} = 7.189051095 \text{ s} = \boxed{7.2 \text{ s}}\end{aligned}$$

6. A 3.00 kg mass, m_1 , slides up a ramp. The angle for the ramp is 28.0° . The 3.00 kg mass is connected to a second mass, m_2 , of 3.25 kg as shown by a light string with a frictionless pulley, &c. Coefficient of kinetic friction is 0.285. Find (a) the acceleration of m_1 , (b) the kinetic energy of m_1 after it has traveled 25.0 cm up the ramp, and (c) the work done on m_1 to move it the 25.0 cm.

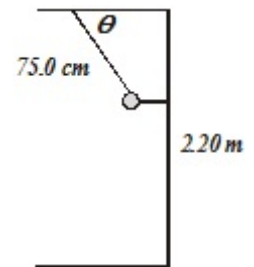


a. $F_{\text{netsys}} = w_2 - w_1 \sin(28.0^\circ) - F_{\text{fric1}} = m_2 g - m_1 g \sin(28.0^\circ) - \mu F_N$
 $F_{\text{netsys}} = m_2 g - m_1 g \sin(28.0^\circ) - \mu \cdot m_1 g \cos(28.0^\circ) = 10.64931817 \text{ N}$
 $a_{\text{sys}} = F_{\text{sys}} / m_{\text{sys}} = 10.64931817 \text{ N} / 6.25 \text{ kg} = 1.703890908 \text{ m/s}^2 = \boxed{1.70 \text{ m/s}^2}$

b. $v^2 = v_i^2 + 2ad$
 $v^2 = 0^2 + 2 \cdot 1.703890908 \text{ m/s}^2 \cdot 0.25 \text{ m} = 0.8519454539 \text{ m}^2/\text{s}^2$
 $\text{KE} = \frac{1}{2} m v^2 = 0.5 \cdot 3 \text{ kg} \cdot 0.8519454539 \text{ m}^2/\text{s}^2 = 1.277918181 \text{ J} = \boxed{1.28 \text{ J}}$

c. $W_{\text{total}} = \text{KE} + W_{\text{fric}} = \text{KE} + F_{\text{fric}} \cdot d = \text{KE} + \mu F_N \cdot d$
 $= 1.277918181 \text{ J} + 0.285 \cdot 3 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot \cos(28.0^\circ) \cdot 0.25 \text{ m} = 3.127472651 \text{ J} = \boxed{3.13 \text{ J}}$

7. A 3.12 kg iron ball is suspended from the ceiling of a room by two cords as shown in the drawing. The ceiling is 2.20 m above the deck. The angle θ is 61.0° . The other cord is perfectly horizontal. Find (a) The tension in both strings. (b) The potential energy of the ball relative to the deck. (c) if the horizontal spring were to break, what would be the velocity of the ball when the other cord is vertical?



a. $w = 3.12 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 30.576 \text{ N}$
 $T_{1y} = T_1 \sin(61.0^\circ) = w$
 $T_1 = 30.576 \text{ N} / \sin(61.0^\circ) = 34.95919398 \text{ N} = \boxed{35.0 \text{ N}}$
 $T_2 = T_{1x} = T_1 \cos(61.0^\circ) = 34.95919398 \text{ N} \cdot \cos(61.0^\circ) = 16.94855356 \text{ N} = \boxed{16.9 \text{ N}}$

b. $d_{y \text{ from ceiling}} = 0.75 \text{ m} \cdot \sin(61.0^\circ) = 0.6559647804 \text{ m}$
 $h = 2.20 \text{ m} - 0.6559647804 \text{ m} = 1.54403522 \text{ m}$
 $\text{PE} = mgh = 3.12 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 1.54403522 \text{ m} = 47.21042088 \text{ J} = \boxed{47.2 \text{ J}}$

c. $\Delta \text{KE} = \Delta \text{PE}$
 $\frac{1}{2} m (\Delta v)^2 = mg \Delta h$
 $0.5 \cdot v^2 = 9.8 \text{ m/s}^2 \cdot (1.54403522 \text{ m} - 1.45 \text{ m})$
 $v^2 = 1.843090312 \text{ m}^2/\text{s}^2$
 $v = 1.357604623 \text{ m/s} = \boxed{1.36 \text{ m/s}}$

8. A 47.0 kg projectile is launched with an initial speed of 72.0 m/s and an angle of 39.8° above the horizontal. The projectile lands on a hillside 7.15 s later. Neglect air friction. (a) What is the projectile's kinetic energy at the highest point of its trajectory? (b) What is the height of the impact point? (c) What is its total energy just before it hits the hillside?

a. $v \text{ at highest point} = v_{\text{horiz}} = v \cdot \cos(39.8^\circ) = 72.0 \text{ m/s} \cdot \cos(39.8^\circ) = 55.3164137 \text{ m/s}$
 $\text{KE} = \frac{1}{2} m v^2 = 0.5 \cdot 47.0 \text{ kg} \cdot (55.3164137 \text{ m/s})^2 = 71907.78218 \text{ J} = \boxed{71.9 \text{ kJ}}$

b. $v_y = v \cdot \sin(39.8^\circ) = 72.0 \text{ m/s} \cdot \sin(39.8^\circ) = 46.08789836 \text{ m/s}$
 $d_y = d_{yi} + v_{yi} t + \frac{1}{2} a t^2$
 $= 0 \text{ m} + 46.08789836 \text{ m/s} (7.15 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(7.15 \text{ s})^2 = 79.02822329 \text{ m} = \boxed{79.0 \text{ m}}$

c. $\sum E_f = \sum E_i$
 $\sum E_f = \frac{1}{2} m v_i^2 = \frac{1}{2} 47.0 \text{ kg} (72.0 \text{ m/s})^2 = 121824 \text{ J} = \boxed{122 \text{ kJ}}$